

A simple entropy-based method for discharges measurements in gauged and ungauged river sites: the case study of coastal Algerian rivers

Une approche simple basée sur l'entropie pour la mesure des débits dans les cours d'eau jaugés et non jaugés, cas des rivières côtières Algériennes

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Abstract. In river monitoring, it is quite difficult to sample velocity points during high floods, especially in the lower portion of flow area, due to the danger that operators might face during the measurement. In this context an important contribution is provided by the entropy theory which identified a linear relationship between the mean flow velocity and the maximum flow velocity which can be easily sampled also during high floods for its position in the upper portion of the flow area. The entropic relationship is robust and based on the estimation of a sole parameter (M). Therefore, if one was able to estimate the dependence of (M) on hydraulic river characteristics and/or morphological basin ones then it would be possible to assess the mean flow velocity and, hence, the discharge just by sampling the maximum velocity through, for instance, non-contact radar sensors, for ungauged basins or weakly gauged ones. Based on the above insights, the purpose of this work is twofold. The first, determine simple formulations between the entropy parameter (M) and morphological characteristics of the basin, subtended from the river site of interest, in terms of drainage density, drainage area and perimeter...etc. The second, identify a formulation between hydraulic river characteristics (Manning's roughness, hydraulic slope,... etc) and the entropy parameter. Algerian gauged river sites are used as case study. That relationship to find M parameters for any sites within the basin was identified with a good correlation of 0.7 and a Mean Square Error (MSE) of 0.046. A quick validation was made as well on some other hydrometric stations in order to check the accuracy of the obtained model. A good representativity of the model was found with errors not exceeding 5% except for ALTAIRAC station (errors of 10%) which, however, need additional analysis. Five other Italian sites were used to check the model validity.

Keywords: Entropy, discharge monitoring, ungauged sites, coastal Algiers watershed.

Résumé. Dans les mesures hydrométriques, il est très difficile de faire un échantillonnage de vitesses durant un événement de crue surtout au niveau des couches proches du fond du cours d'eau, du fait du risque encouru par les opérateurs lors des mesures. Afin de pallier à cette insuffisance, la théorie d'entropie a permis de développer une relation linéaire entre les vitesses moyenne et maximale de l'écoulement qui peut être, aisément, mesurée même en période de fortes crues du fait de sa position dans la partie supérieure de l'écoulement. La relation entropique est robuste et basée sur l'estimation du paramètre d'entropie (M). Si on peut démontrer qu'il existe une relation entre (M) et les caractéristiques du bassin drainé, donc, il sera possible de déterminer la vitesse moyenne et le débit seulement par la mesure de la vitesse maximale (par exemple par un radar non intrusif) même pour les bassins non jaugés ou faiblement jaugés. Le but de ce travail a un double objectif. Le premier concerne la détermination d'une simple relation entre le paramètre d'entropie (M) et les caractéristiques morphologiques du bassin concerné (densité de drainage, surface drainée, etc...). Le second est d'identifier une expression entre le paramètre d'entropie (M) et les caractéristiques hydrauliques du cours d'eau (Coefficient de Manning, pente hydraulique ...). Des rivières jaugées algériennes ont été choisies comme exemple pour cette étude. La relation pour estimer le paramètre M, pour tout site à l'intérieur d'un bassin donné, a été développée avec un coefficient de corrélation de 0,70 et une Moyenne des Carrés des Ecartés (MCE) de 0,046. Une validation rapide a été entreprise pour vérifier la robustesse du modèle en utilisant les mesures de stations hydrométriques non considérées dans la phase de production du modèle. Cette validation a montré que les erreurs se situent au dessous de 5%, mis à part la station ALTAIRAC, où l'erreur est de l'ordre de 10%, ce qui nécessite une analyse plus approfondie. Cinq stations italiennes ont été utilisées pour vérifier la validité du modèle.

Mots clés: Entropie, mesure des débits, sites non jaugés, bassin des côtiers algérois.

INTRODUCTION

Usually in hydrological practice, discharges measurements are generally addressed using standard methods based especially on the velocity field exploration. The velocity profiles in different verticals are obtained by sampling three or more velocity points, but sometimes during high floods it is very tough to conduct the sampling velocity points especially in the lower part of the flow. Several measurements techniques exists but most of them are based on velocity field measurements which take a lot of time and efforts and are difficult to carry out during high flood due to high velocity and floating material (Chiu 1999). Recently, the introduction of Acoustic Doppler Current Profilers (ADCP) installed on moving-vessels has allowed operators to address the above mentioned problems even if several limitations remain. Indeed, during high floods any increase in sediment transport produces a reduction of the signal-noise ratio of acoustic sensors (Corato *et al.* 2014), Moreover, there are some methods based on the sampling of one, two or three points of each vertical were developed (Ardiclioglu *et al.* 2007). These approaches are based on the turbulent boundary layer theory (Fenton 2002) based on the logarithmic law of the velocity distribution, or the Prandtl Von Karman law (Chow 1959). The sampling difficulties during high floods reduces considerably their application and, hence, the accuracy of the discharge measurements (Chiu & Chen 1999). According to Singh (2013), the empirical velocity distribution models are not capable of incorporating the velocity uncertainties. Some modern approaches, mainly based on the concept of entropy, as in the fields of hydraulics and hydrology, consider velocity as a probabilistic variable, taking this uncertainty into account.

For these reasons and others, the *Entropy* method developed by Chiu (1987) was tested at some gage sites of the Coastal Algiers Watershed (Ammari & Remini 2010), and it gave reliable results in terms of discharges and flow area assessment as well. This method can estimate the discharge with the maximum velocity sampling (Chen 2014) which is multiplied by the flow area modeled using the entropy theory for bathymetry if topographic data are not available (Moramarco *et al.* 2013). A first approach was proposed by Moramarco *et al.* (2008) in order to estimate the entropy parameter, M , as a function of Rosgen river classification (Rosgen 1994). Further works were conducted to describe the *entropy* parameter in terms of Froude number (Mirauda *et al.* 2011) and Manning roughness (Greco *et al.* 2014), even though their practical use in flow measurements are limited because of the large variability of Froude number or Manning roughness in natural streams.

In this paper we are first interested to investigate the relationship between M and hydraulic and geometric characteristics of river cross-section like

hydraulic radius, hydraulic slope and manning roughness, using the *entropy* parameters and other fast measured ones. Then, the analysis is extended for ungauged river sites trying to develop a simple regression model to estimate the *Entropy* parameter as a function of some watershed characteristics.

MATERIALS AND METHODS

Based on the concept of *entropy*, Chiu developed a simple expression, which can describe a possible velocity profile (Chiu 1987, 1988, 1989):

$$u = \frac{U_{max}}{M} \ln \left[1 + (e^M - 1) \frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \quad (1)$$

where u is the horizontal velocity, $\frac{\xi - \xi_0}{\xi_{max} - \xi_0}$ represents the cumulative probability distribution function, in which ξ is a function of the spatial coordinates in the physical space; $\xi_{max} = \xi$ at the point where the maximum velocity, u_{max} occurs; $\xi_0 = \xi$ at the channel bed where $u=0$, is the entropic parameter. The appealing aspect behind this theory is that the mean flow velocity, u_m , can be easily assessed (Chiu 1988) as:

$$u_m = \frac{Q}{A} = \Phi(M) u_{max} \quad (2)$$

u_{max} is the maximum velocity, $\Phi(M)$ is a function of the *entropy* parameter M deduced after *entropy* maximization (Chiu & Hsu 2006).

$$\Phi(M) = \frac{U_m}{U_{max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} \quad (3)$$

If $\Phi(M)$ is constant for the same gage site, therefore M is also and can describe the velocity profiles according to the Eq. (1)

Let's consider the well-known Manning Equation

$$Q = \frac{\sqrt{S_f}}{n} AR^{\frac{2}{3}} \quad (4)$$

With S_f , energy slope; n , Manning's roughness; A , flow area and R , hydraulic radius.

Using the *entropy* method, we can turn the last equation into:

$$\alpha = \frac{\sqrt{S_f}}{n} = \frac{Q}{AR^{\frac{2}{3}}} = \frac{u_m A}{AR^{\frac{2}{3}}} = \frac{u_m}{R^{\frac{2}{3}}} = \frac{\Phi(M) u_{max}}{R^{\frac{2}{3}}} = \frac{\Phi(M) u_{max}}{h_m^{\frac{2}{3}}} \quad (5)$$

For large flow depths, the hydraulic radius approaches the hydraulic depth ($h_m=A/T$), where T is the top width of the channel.

Therefore, if a good topographical survey of the river site is available (Ammari & Remini 2010), it will be theoretically easy to define the ratio between the square root of the energy slope and the manning

roughness by exploiting M in Eq.(3), once u_{max} and h_m (maximum velocity and mean depth) are known. Therefore, once α is estimated by M , and considering that α is constant for high stage, Eq.(5) allows to estimate the extrapolate the rating curve at river site just knowing the conveyance $AR^{\frac{2}{3}}$.

Estimate of M based on watershed characteristics

For ungauged river sites, the lacking of velocity dataset does not allow to estimate the entropy parameter M . For that, the dependence of M from watershed characteristics is investigated here, in order to get a simple direct relationship to be applied at ungauged sites. Over the Coastal Algiers Watershed we have many important ungauged streams, like Damous stream on which a Dam of $125Mm^3$ is in construction, this example of inexistent data have a real impact on the management of the water resources especially in region of water scarcity.

Using the available data, we tried to develop a simple correlative expression which help to find a correct value of the *entropy* parameter (and so the ratio $\Phi(M)$) in order to estimate quickly the mean velocity as a function of the maximum velocity, which is easy to locate and to measure (Ammari 2010) and having a topographical survey of cross-section a robust estimation of discharge is expected. M values of the investigated gage sites are used to identify the potential relationship between M and well known characteristics of the drained basin of each section. The following main watershed characteristics are used for the analysis: Area (A), perimeter (P), mean altitude (H_m), slope index (I), drainage density (D_d), concentration time (T_c) and torrentiality coefficient (C_t).

The adopted relationship is thus defined:

$$M = \beta_1 A + \beta_2 P + \beta_3 D_d + \beta_4 C_t + \beta_5 I + \beta_6 H_m + \beta_7 T_c \quad (6)$$

$\beta_i, i=1, \dots, 7$ Regression parameters

Study zone

The coastal Algiers watershed (Fig. 1) is an extended watershed from the East (Bejaia) to the West (Mostaganem) of Algeria, covering an area of 11972 km^2 with an average length of 24 km and a 500km long, this wide extension gives it a great climatic, hydrologic, geomorphologic variability. It's limited to the North by the Mediterranean Sea, in the South by the Soummam, Isser and Cheliff watershed which also borders it from the West. In the East we find the Constantine's watershed.

The monitoring network is not representative of hydrometric conditions of coastal Algiers watershed because of few hydrometric stations; while the National Water Resources Agency aims to improve its monitoring network by the equipment of old stations and news ones in some ungauged sites. Topography of the watershed is characterized by an important mountainous chains which are considered as a southern border, called Tellian Atlas where altitudes exceed 1000 m, like Chrea (1550 m at the "retrait") and Djurdjura (2308 m at Lalla Khedidja). All streams are fed by springs in those mountains. This watershed is well drained with many streams especially in the East over the Sebaou Sub-watershed. Most part of streams are controlled by fixed or mobile gauge stations, but the data collected presents large blank periods, even during high floods, where the flow is so important that it is difficult to conduct

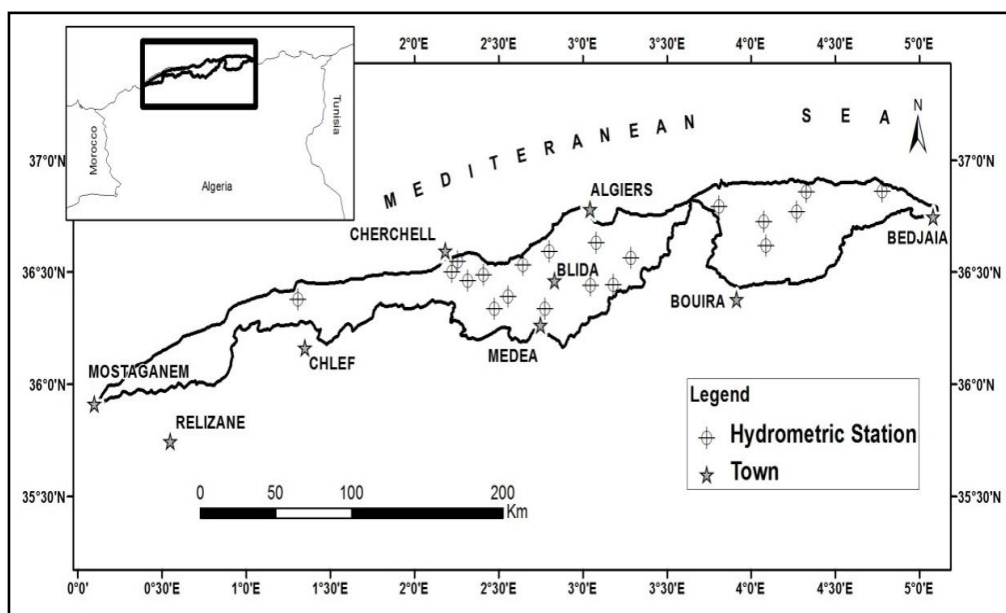


Figure 1. Coastal Algiers watershed situation.

Table 1. Flow characteristics at gage sites. N, number of measurements, Q, discharge and D flow depth.

Sub-watershed	Station	N	Q (m ³ /s)	D (m)
Oued Sebaou Maritime	Baghlia	121	0.01 - 173.12	0.1 - 6.65
Oued Aissi	RN 30	185	0.01 - 13.82	0.1 - 1.78
Oued Sebaou Rabta	Freha	74	0.022 - 2.82	0.1 - 0.7
Oued El Hrarrach Maritime	Baraki	125	0.6 - 9.71	0.15-0.55
Oued El Hrarrach Maritime	Gorges de larbaa	141	0.06 - 7.56	0.1- 0.65
Oued El Hrarrach Amont	Rocher des Pigeons	300	0.08 - 14.05	0.1 -0.98
Oued Mazafran	Fer à Cheval	317	0.014 - 349.60	0.1 - 7.8
Oued Chiffa	Amont des Gorges	166	0.015 - 10.92	0.1 - 1.1
Oued Djer-Bouroumi	Attatba	117	0.011 - 4.26	0.1 - 0.85
Oued Djer-Bouroumi	El Affroun	98	0.01 - 3.17	0.1 - 0.85
Oued Djer-Bouroumi	Boumedfaa	63	0.035 - 21.3	0.1 - 0.85
Côtiers Cherchell	Hadjout	37	0.01-1.11	0.1-0.93
Côtier Cap Matifou	Ouled Ali	128	0.03 - 6.57	0.1- 0.7
Côtiers Cherchell	Pont RN11	53	0.08 - 5.30	0.13 - 2.04
Côtiers Cherchell	Borj Ghobrini	52	0.036 - 15.96	0.1- 0.95
Côtiers Cherchell	Mesdour	111	0.2- 3.08	0.13-0.45
Côtier Tenes	Sidi Akacha	115	0.03 - 7.8	0.1 - 0.85
Tigzirt	Azzefoun	52	0.06-1.74	0.3-0.7
Harezza	El Ababsa	68	0.16-11.83	0.1-0.85
Hamiz	Pont D9	183	0.06-33.81	0.1-1.05
Reghaia	Reghaia	330	0.07-3.12	0.1-0.67
Semar	Altairac	520	0.06-37.80	0.34-1.6

traditional measuring methods like the sampling of velocity fields by current meters. So the National Hydraulic resources agency uses other techniques like rating curves and floating bodies, but the methods failed to measure high discharges during floods where the volume of water is more than 60% of the one drained by streams with more than 80% of the sediments transport. Therefore, the necessity to develop new methods to optimize time and money in order to get more data with a good accuracy especially during high floods is recommended.

The *Entropy* method could be, therefore, useful to get more data with minimum time and effort and can be used for automatic gauge stations having continuous records.

RESULTS AND DISCUSSION

The velocity dataset was collected by the National Water Resources Agency covering a period from 1990 to 2010 of the Algiers Watershed (Fig. 1). Dataset is widely different in terms of water levels and discharges. The first remark here is the fact most of the data are limited to low flow, showing the difficulties to conduct a lot of measurements under high floods conditions. The flow characteristics are illustrated into the table 1. After developing the model based on the eighteen Hydrometric site's data,

we used the four stations in bold in the table 1 to check the ability of the model to estimate a correct values of the entropy parameter.

Estimate of M parameter and robustness analysis

Figure 2 illustrates a good relationship between the mean and maximum velocities, and $\Phi(M)$ can be considered constant for all the gauge stations investigated, confirming the results of Chiu (1989, 2006) and Moramarco *et al.* (2004, 2010).

Therefore, M deduced by equation (3) can be considered as a parameter characteristic of the gage river site with a coefficient of determination, R^2 , greater than, on average, 0.94.

Correlation analysis

Eq.6 is investigated by addressing a sensitivity analysis on some watershed characteristics which may influence the M estimate by using the XIStat code (Addinsoft 2014). The correlation allows to estimate the normalized Beta coefficients of a linear regression (Montgomery & Runger 2011) permitting the possibility to understand which watershed characteristics are more influent in the M estimation. It is worth noting that the lower Beta coefficient, the lower the influence of the watershed characteristic.

All results are summarized in the table 2 as well.

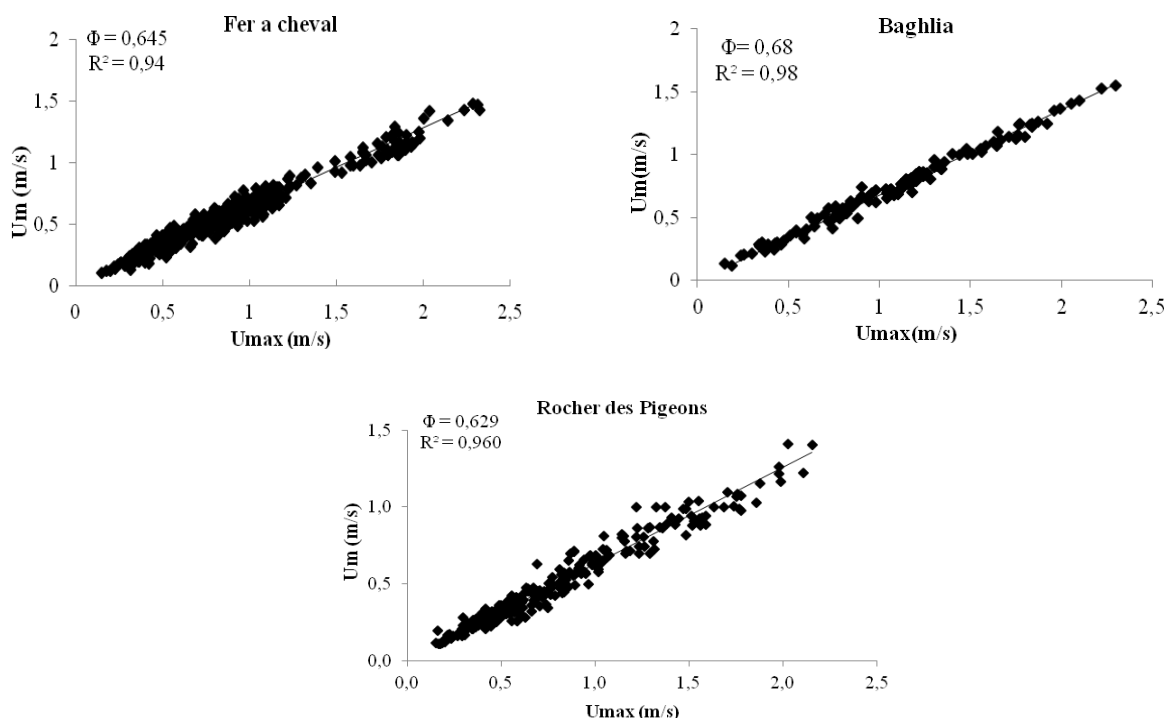


Figure 2. Mean / Max velocity relationship for three gauge stations (Ammari 2012).

Table 2. Entropy parameter M and the ratio $\Phi(M)$;mean/maximum velocities.

N°	Gauge Station	Wadi	$\Phi(M)$	M	R ²
1	Baghlia	Sebaou	0.681	2.36	0.98
2	Hadjout	Bourkika	0.636	1.71	0.94
3	RN30	Aissi	0.641	1.78	0.97
4	Freha	Dis	0.662	2.08	0.95
5	Gorges de Larbaa	Djemmaa	0.627	1.59	0.98
6	BARAKI	El harrach	0.627	1.59	0.96
7	Rocher des pigeons	El harrach	0.629	1.61	0.96
8	Fer à cheval	Mazafran	0.645	1.84	0.94
9	Amont des gorges	Chiffa	0.641	1.79	0.97
10	Attatba	Bouroumi	0.613	1.4	0.93
11	El affroun	Djer	0.588	1.07	0.92
12	Boumedfaa	Djer	0.587	1.06	0.96
13	Azeffoun (RN24)	M'letat	0.673	2.24	0.94
14	Ouled Ali	Barek	0.619	1.48	0.97
15	Pont RN11	Belah	0.659	2.03	0.97
16	Bordj ghoibrini	El hachem	0.627	1.58	0.95
17	Mesdour	Boukdir	0.640	1.77	0.94
18	Sidi Akacha	Allalah	0.648	1.88	0.94
19	El Ababsa	Harezza	0.645	1.97	0.96
20	Pont D9	Hamiz	0.620	1.17	0.94
21	Reghaia	Reghaia	0.634	1.69	0.82
22	Altairac	Semar	0.677	2.25	0.95

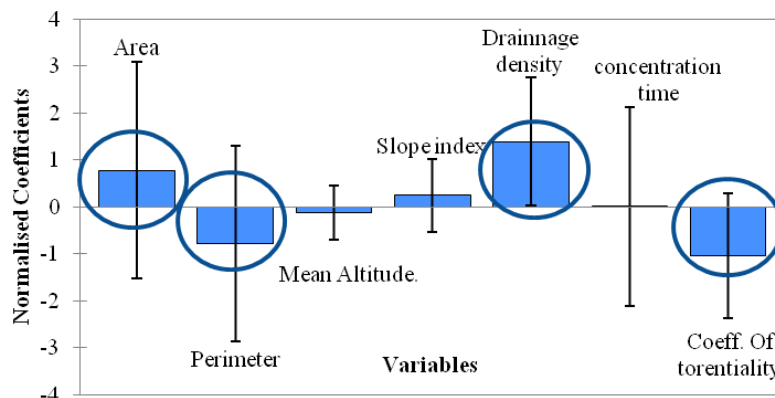


Figure 3. 1st Run; Normalized coefficients (Int. conf. 95%).

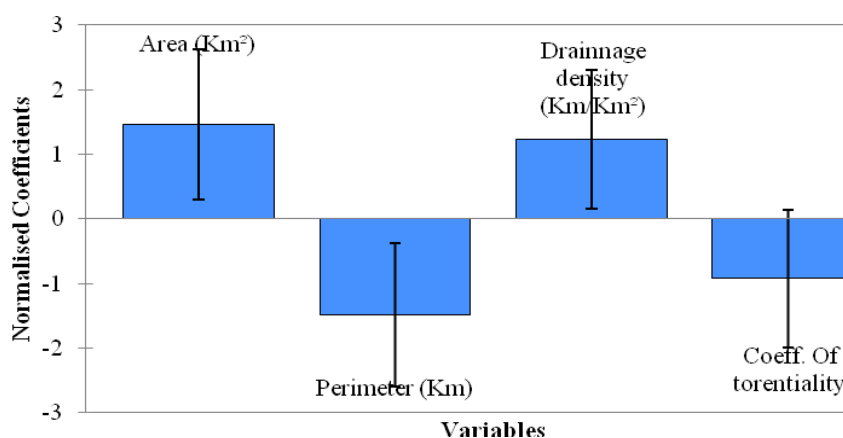


Figure 4. 2nd Run , Normalized coefficients (Int. conf. 95%).

We observe that the Mean Altitude, slope index and concentration time have not a great influence on the *entropy* parameter, so we run a new correlation with the other four (04) characteristics

Therefore, the correlation is investigated as a function of four drained basin characteristics with a satisfactory partial correlation coefficient and total correlation coefficient according to the traditional statistical tests (Student and Fisher).

$$M = 1.207 + 7.753 * 10^{-4}A - 9.873 * 10^{-3}P + 0.405D_d - 1.61 * 10^{-2}C_t \quad (7)$$

The model is obtained with a coefficient of determination R^2 of 0.70 and MSE (Mean Square Error) of 0.046.

The relationship between real and predicted M values are illustrated in the figure 5.

It is observed that all points are in the direct vicinity of the straight line of slope equal to the, unity, and into the confidence interval. Also in order to validate the model, we consider the four river sites used for the validation, ALTAIRAC, ABABSA, REGHAIA and PONT D9.

We observe that only ALTAIRAC is out of the confidence interval, all three other stations are inside. In order to test the reliability of Eq.(7), figure 7 shows errors in percentage in estimating $\Phi(M)$, using Eq.(3) at investigated gage sites. The estimated $\Phi(M)$ values are less than 5% of the actual values for the training gage sites. For the validation, the error exceeds 5% only for ALTAIRAC, which need more investigation in terms of reliability of velocity measurements. Therefore, Eq.(7) can describe the variation of the entropy parameter over the whole watershed, for all stations in function of their morphological characteristics. Finally, to check further the robustness of the method five Italian gage sites are used for the analysis. Their main watershed characteristics are detailed in table 3. Figure 8 shows results in terms of percentage errors in estimating M. It is observed that four sites out the five Italian gage sites show an error greater than 10%, but less than 20% and this is a satisfactory result considering that no velocity dataset is used for the analysis of Italian sites. However, let's try now to see what happens if two Italian gage sites (S. Lucia and P. Felcino) are included in the dataset used to identify the prediction model, while the last three are used for the validation (M. Molino, Rosciano, P. Nuovo).

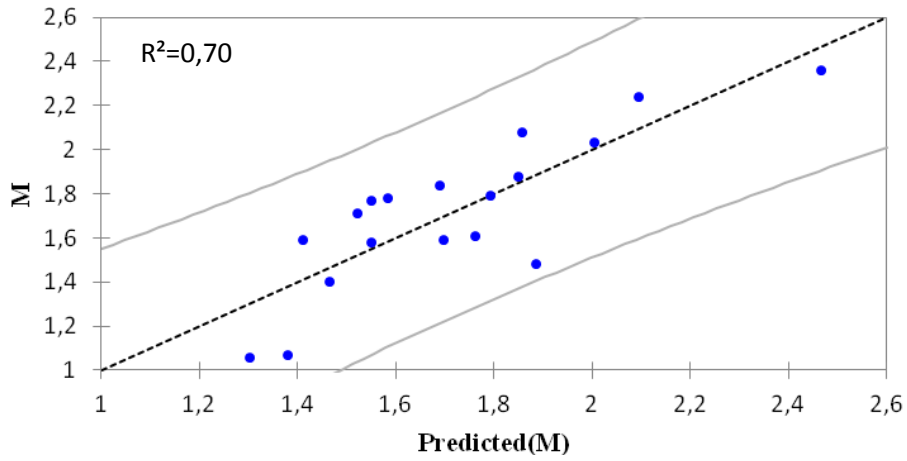


Figure 5. Real /Predicted M values (conf. inter 95%).

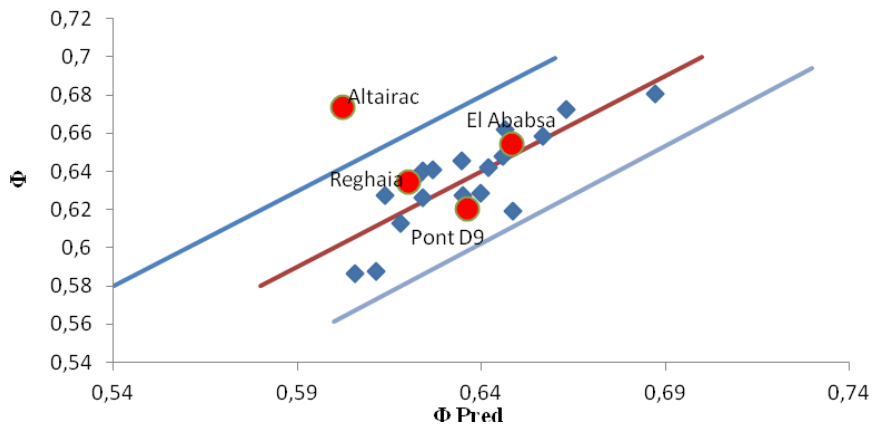


Figure 6. Real/Predicted Φ values (conf. inter 95%) Validation.

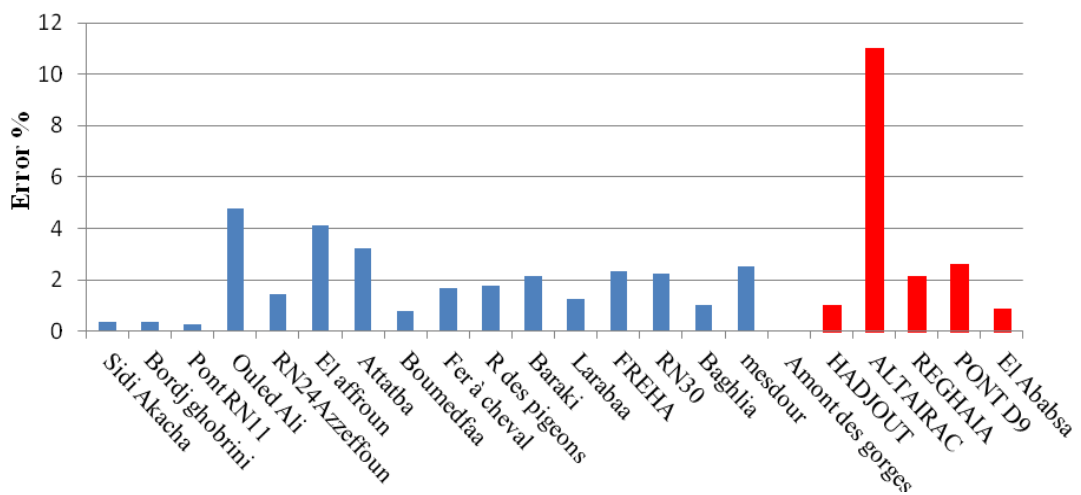


Figure 7. Percentage error in $\Phi(M)$ assessment.

Table 3. Main characteristics of italian gage sites. For symbols see text.

N°	Gauge Station	River	Φ(M)	M	R ²	A(km ²)	P(km)	Dd (km/km ²)	Ct
1	St. Lucia	Tiber	0,668	2,16	0,99	933	173,5	1,57	1,87
2	P. Felcino	Tiber	0,668	2,17	0,99	2039	261,5	1,57	1,94
3	P. Nuovo	Tiber	0,662	2,08	0,98	4134,5	442,1	1,52	1,75
4	M. Molino	Tiber	0,640	1,77	0,98	5268,6	489,7	1,54	1,79
5	Rosciano	Chiascio	0,669	2,18	0,99	1944,5	265,1	1,48	1,6

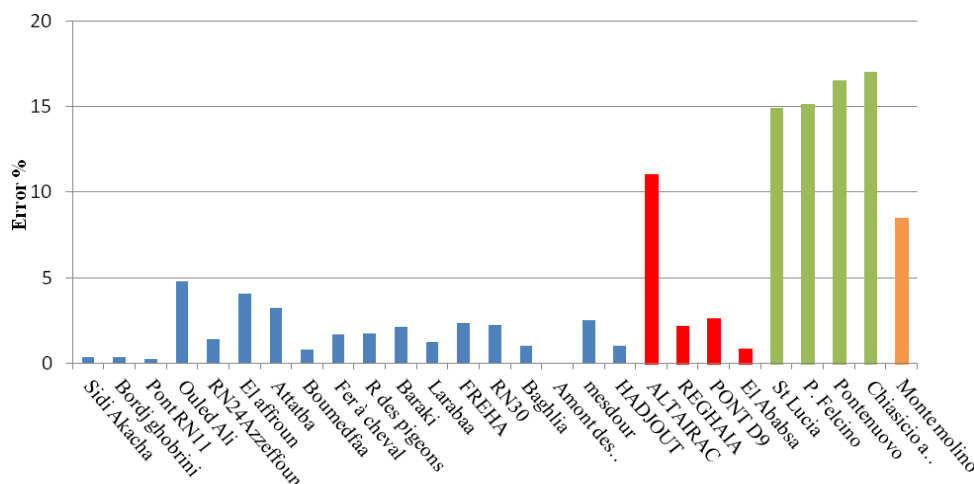


Figure 8. 2nd Φ(M) estimated errors.

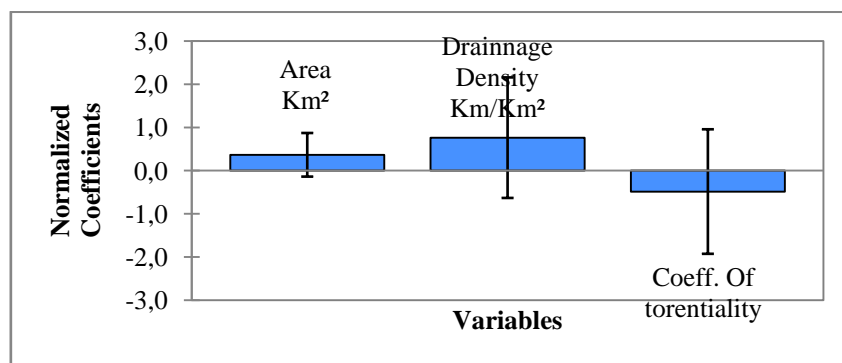


Figure 9. As in figure 3, but including two Italian river sites.

When we add the Italian sites we observe that only three characteristics influence the analysis of M. By applying the XLStat algorithm (Addinsoft 2014) the new equation is inferred:

$$M = 1.11 + 1.842 * 10^{-4}A + 0.232D_d - 8.29 * 10^{-3}C_t \quad (8)$$

Including the three Italian sites in the production of the model, we observe that there are only three parameters remaining (Fig. 9), so we will see below if this model (including algerian and italian sites) is a robust one to estimate entropy parameter for all investigated sites. As shown in figure 10, all the stations except ALTAIRAC (cited in the upper part

of the paper) are within the interval of confidence. Errors are less than 10% as shown in figure 11, which is more than acceptable if historical velocity data are missing. As regards the analysis on the α parameter as a function of M, Eq.(5), figure 12 depicts the comparison between α versus the actual values and as can be seen the comparison is really good. Therefore a simple measured flow section parameters (total flow section, top width or maximum depth) can be used to determine the ratio between the square root of hydraulic slope and the Manning Roughness using the entropy parameter obtained by historical gauge date or by Eq.(8) for an ungauged site.

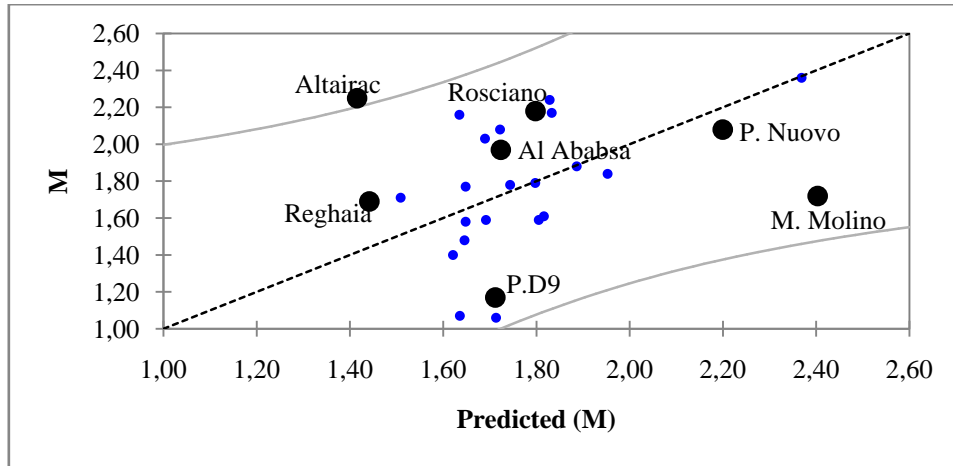


Figure 10. Comparison between Observed and Computed M by Eq.(8) including the two italian sites in the analyzed dataset.

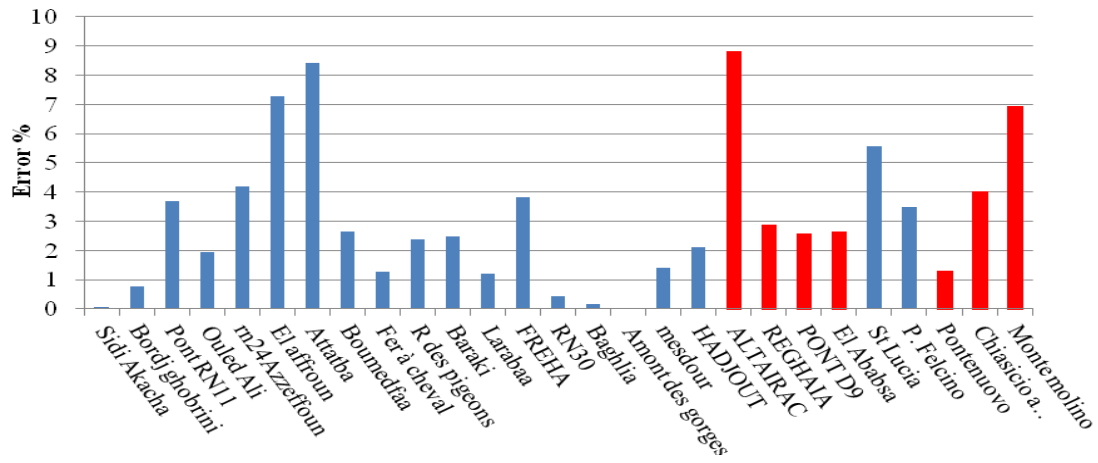


Figure 11. 3rd $\Phi(M)$ estimated errors.

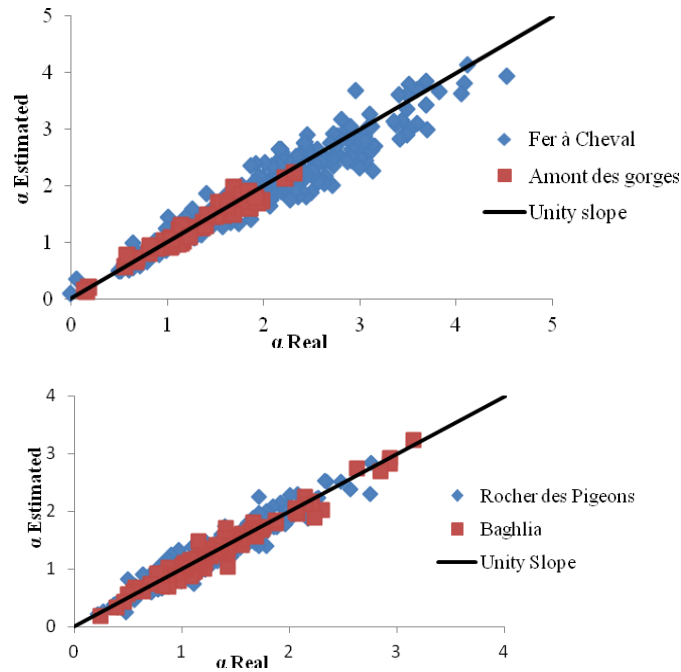


Figure 12. Real /Entropy estimated values of α parameter.

CONCLUSION

The *Entropy* method turned out very useful in estimating the mean flow velocity by sampling the maximum velocity only, as shown through several studies published last two decades. The Entropy approach can be conveniently adopted to carry out quick measurements and without dangers for the operators during high floods.

For ungauged sites, historical data are not available or not sufficient, and the simple linear relationship proposed here to estimate the M value as a function of some drained basin characteristics turned out very useful and robust for the purpose. The linear relationship was validated in several gage river sites with a maximum error less than 5%, except for ALTAIRAC station. Therefore the proposed relationship would allow to use only the measured maximum velocity, occur in the deepest vertical according to Ammari (2012) and deduce the mean velocity from which the discharge can be estimated considering the observed flow area during the measurement. The application of method to five Italian sites proved further its reliability.

The determination of the (α) parameter is of considerable interest in river monitoring, because it associates the most important hydraulics characteristics, i.e., hydraulic slope and Manning roughness. The study will go on, using additional hydrometrical stations data from the Algiers watershed and from neighboring ones. These results may be useful also in flood monitoring and routing to find flow characteristics in several sections along the path of the flood wave.

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